Midterm Evaluation Problem and Answer Sheets

Section:_____ Name: _____

Write your answer **in the box** at the beginning of each problem.

1. ☐ What amount of a 60% acid solution must be mixed with a 30% solution to produce 300 mL of a 50% solution?
(A) 100 mL (B) 125 mL (C) 150 mL (D) 175 mL (E) 200 mL. E

2. ☐ If a manufacturing sells x units of a certain product, revenue R and cost C (in dollars) are given by

$R = 20x$

$C = 2000+8x+0.0025\ x^2$

Use the fact that profit= revenue-cost, to determine how many units the manufacturer should sell to enjoy a profit of at least $2400.
(A) between 400 and 1400 units (B) between 400 and 2400 units (C) between 400 and 3400 units (D) between 400 and 4400 units (E) between 400 and 5400 units. D

3. ☐ Find the area of the region that lies outside the circle $x^2 + y^2 = 4$ but inside the circle $x^2 + y^2 - 4y - 12 = 0$.
(A) 8 π (B) 10 π (C) 12 π (D) 14 π (E) 16 π . C

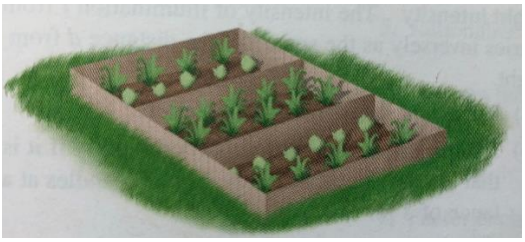
4. ☐ Find the area of the triangle formed by the coordinate axes and the line $2y + 3x - 6 = 0$.
(A) 2 (B) 4 (C) 6 (D) 8 (E) none of the above. E

5. ☐ A small business buys a computer for $2400. After 4 years the value of the computer is expected to be $200. For accounting purposes the business uses linear depreciation to assess the value of the computer at a given time t. This means that if V is the value of the computer at time t, then a linear equation is used to relate V and t. Find the depreciated value of the computer 3 years from the date of purchase.
(A) $750 (B) $850 (C) $950 (D) $1050 (E) $1150. A

6. ☐ The lift of an airplane wing at takeoff varies jointly as the square of the speed S of the plane and the area A of its wings. A plane with a wing area of 50 m^2 traveling at 80 km/h experiences a lift of 6400 N. How much lift would a plane with a wing area of 60 m^2 traveling at 80 km/h experience?
(A) 7280 N (B) 7480 N (C) 7680 N (D) 7880 N (E) 8080 N. C

7. ☐ Antony leaves Kingstown at 2:00 P.M. and drives to Queensville, 256 km distant, at 72 km/h. At 2:15 P.M. Helen leaves Queensville and drives to Kingstown at 64 km/h. At what time do they pass each other on the road?
(A) 3:45 P.M. (B) 4:00 P.M. (C) 4:15 P.M. (D) 4:30 P.M. (E) 4:45 P.M. B

8. ☐ A homeowner wishes to fence in three adjoining garden plots, one for each her children, as shown in the figure. If each plot is to be 80 m^2 in area and she has 88m of fencing material at hand, what is the possible dimension should each plot have?
(A) 4 x 20 (B) 5 x 16 (C) 6 x (80/6) (D) 8 x 10 (E) none of the above. D



9. ☐ A function is given.

$$f(x) = 1 - 3x^2; x = 2, x = 2 + h$$

What is the average rate of change between the given values of the variable?
(A) -3h - 12 (B) -3h + 12 (C) 3h - 12 (D) 3h + 12 (E) none of the above. A

10. ☐ A function is given.

$$f(x) = x^2,$$

and the indicated transformations are applied to its graph in the given order: stretch vertically by a factor of 2, shifted downward 2 units, and shit 3 units to the right. What is the equation after the final transformation?
(A) $y = -2(x+3)^2 + 2$ (B) $y = 2(x+3)^2 + 2$ (C) $y = 2(x-3)^2 - 2$ (D) $y = -2(x-3)^2 - 2$ (E) none of the above. C

11. ☐ Use the following functions: $f(x) = 2x - 3$ and $g(x) = 4 - x^2$,
to evaluate the composition of functions (g ◦ f)(-2)
(A) 54 (B) -54 (C) 45 (D) -45 (E) none of the above. D

12. ☐ A function is given. $f(x) = \frac{x-2}{x+2}$
What is the inverse function of f(x)?
(A) $f^{-1}(x) = \frac{-2(x+1)}{x-1}$ (B) $f^{-1}(x) = \frac{2(x+1)}{x-1}$ (C) $f^{-1}(x) = \frac{-2(x-1)}{x+1}$ (D) $f^{-1}(x) = \frac{2(x-1)}{x+1}$ (E) none of the above. A

13. ☐ The population of a planned seaside community in Florida is given by the function

$$P(t) = 3000 + 200t + 0.1t^2,$$

Where t represents the number of years since the community was incorporated in 1985. Find the average rate of change between t = 10 and t = 20.

(A) 173 (B) 183 (C) 193 (D) 203 (E) 213 people/year. D

14. ☐ A stone is thrown upward from the top of a building. Its height (in meters) above the ground after t seconds is given by the function

$$h(t) = -5t^2 + 20t + 10,$$

What maximum height does it reach?

(A) 10 (B) 15 (C) 20 (D) 25 (E) 30 meters. E

15. ☐ Ella is saving for her retirement by making regular deposits into a 401(k) plan. As her salary rises, she finds that she can deposit increasing amounts each year. Between 1995 and 2008 the annual amount (in dollars) that she deposited was given by the function $D(t) = 3500 + 15t^2$, where t represents the year of the deposit measured from the start of the plan (so 1995 corresponds to t = 0, 1996 corresponds to t = 1, and so on). Assuming her deposits continue to be modeled by the function D, in what year will she deposit $17,000?

(A) 2005 (B) 2010 (C) 2015 (D) 2020 (E) 2025 . E

16. ☐ The number of apples produced by each tree in an apple orchard depends on how densely the trees are planted. If n trees are planted on an acre of land, then each tree produce 900-9n apples. So the number of apples produced per acre is

$$A(n) = n(900 - 9n).$$

How many trees should be planted per acre to obtain maximum yield of apples?

(A) 40 (B) 45 (C) 50 (D) 55 (E) 60. C

17. ☐ A rational function is given. $f(x) = \frac{x^4 - x^3 + x^2 - x + 2}{x - 2}$

What is the remainder of f(x) after doing the synthetic division?

(A) -2 (B) 4 (C) 2 (D) 10 (E) 12. E

18. ☐ A polynomial is given.

$$p(x) = x^3 - 3x - 2$$

What are the zeros of the polynomial?

(A) -2, -1 (B) -2, 1 (C) 2, -1 (D) 2, 1 (E) none of above. C

19. ☐ A rational function is given.

$$r(x) = \frac{x^2 + 2x}{x - 1}$$

What is the slant asymptotes of $r(x)$?
(A) x+1 (B) x+2 (C) x-2 (D) x+3 (E) x-3. D

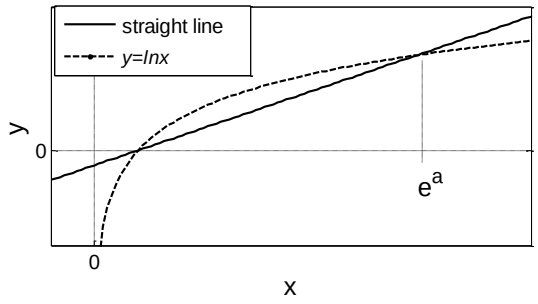
20. ☐ Let $P(x) = 2x^4 - 7x^3 + x^2 - 18x + 3$.

Use Descartes' Rule of Signs to determine how many negative real zeros P can have?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4. A

21. ☐ After simplification, the expression $\log_2 3 - 3\log_2 x + \frac{1}{2}\log_2(x+1)$ can be reduced to (A) $\log_2 3x^3\sqrt{x+1}$ (B) $\log_2 \frac{3\sqrt{x+1}}{x^3}$ (C) $\log_2 3x^3\left(\frac{x+1}{2}\right)$ (D) $\log_2 \frac{3x^3}{\sqrt{x+1}}$ (E) $\log_2\left(3 - x^3 + \sqrt{x+1}\right)$

B

22. ☐ Which of the following represents the straight line intersecting the *ln* function shown below: (A) y=a(x-1) (B) $y = \frac{a}{e^a}(x-1)$ (C) $y = \frac{e^a}{e^a - 1}(x-1)$ (D) $y = \frac{e^a - 1}{a}(x-1)$ (E) $y = \frac{a}{e^a - 1}(x-1)$ E



23. ☐ The value of $\log_9 0.36$, rounded to two decimal places, is about (note: $\log_{10}2$=0.301, $\log_{10}3$=0.477)
(A) 0.30 (B) -0.30 (C) 0.47 (D) -0.46 (E) 0.50 D

24. ☐ Which of the following is the inverse function $f^{-1}(x)$ of $f(x)=\log_2(\log_{10} x)$: (A) 10^{2x} (B) 10^{2^x} (C) $2^x 10^x$ (D) 2^{10^x} (E) 2^{10x} B

25. ☐ The drilling of a jackhammer was measured at 131dB. The sound of whispering was measured at 28dB. (Note that log2=0.301). The ratio of the intensity of the drilling to that of the whispering would be approximately: (A) $2X10^{10}$ (B) $3X10^{103}$ (C) $3X10^{10}$ (D) $2X10^{103}$ (E) $3X10^9$ (note: $dB=10\log I/I_0$) A

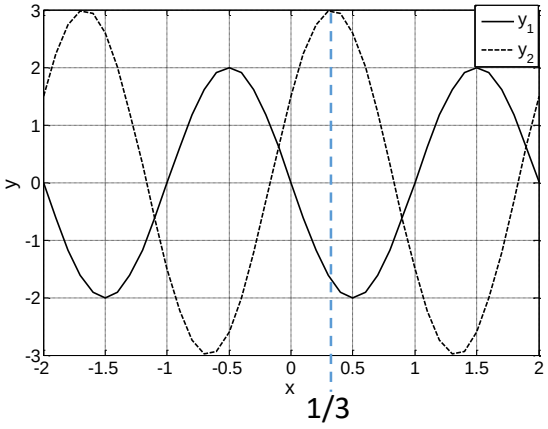
26. ☐ A car engine runs at a temperature of 87°C, when the engine is turned off, it cools according to Newton's Law of Cooling ($T=T_s+D_0e^{-kt}$) with constant k=0.0347, where the time is measured in minutes. How long does it take for the engine to cool to 33°C if the surrounding temperature is 15°C.
(A) 44min (B) 42min (C) 40min (D) 38min (E) 36min (note: ln2=0.6931) C

27. ☐ Simplify the logarithmic expression $\log_2 3 \cdot \log_7 64 \cdot \log_3 5 \cdot \log_5 49$, one would obtain (A) 12 (B) 10 (C) 8 (D) 7 (E) 1 A

28. ☐ A strange bank account offers a 6% annual interest rate, compounded every two months, how long will it take for this account value to double? (A) 6 years (B) 6 years (C) 10 years (D) 12 years (E) 14 years (note: log1.01=0.0043, and compound interest $A=P(1+r/n)^{nt}$) D

29. ☐ The answer to the inequality $\log_{10}(x^2-7x)<\log_{10}(3-x)+\log_{10}2$ is (A) $-1<x<0$ (B) $x>7$ or $x<0$ (C) $x>7$ or $-1<x<6$ (D) $-1<x<6$ (E) $-1<x<3$ A

30. ☐ The graphs of the functions y_1 and y_2 are plotted below, if they are expressed in the standard for of $y=a\cos(kx-\phi)$, what is the phase difference of y_1 relative to y_2? (A) 5/6 lagging (B) 5/6 leading (C) $5\pi/6$ lagging (D) $5\pi/6$ leading (E) none of the above D



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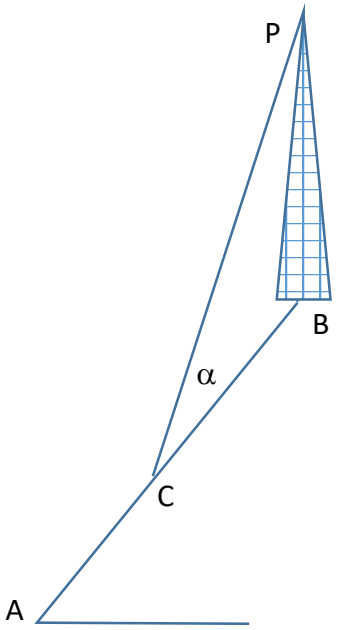
31. ☐ The value of $\sin(\tan^{-1}(-3/4))$ is (A) -0.75 (B) -0.6 (C) -0.8 (D) 0.8 (E) none of the above B

32. ☐ If $\sin\theta+\cos\theta=\frac{1}{\sqrt{2}}$, then which of the following is true (A) θ can be in Quadrant I (B) θ must be in Quadrant II or IV (C) θ must be in Quadrant I or IV (D) θ can be in Quadrant I, II, or III (E) none of the above is correct B

33. ☐ If $0<\theta<45$, find the value of $\sin^2(45°-\theta)+\sin^2(45°+\theta)+\tan(45°-\theta)\tan(45°+\theta)$ (A) 0 (B) 1 (C) 2 (D) -1 (E) -2 C

34. ☐ If a=sin1230°, b=cos(-1150°), c=tan1670°, d=cos2260°, then which of the following ordering is correct? (A) c>a>b>d (B) c>b>a>d (C) a>c>b>d (D) a>b>c>d (E) none of the above A

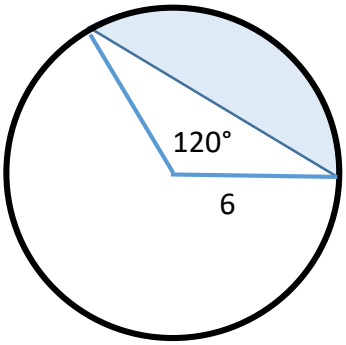
35. ☐ A communications tower is located at the top of a steep hill, as shown at right. The angle of inclination of the hill is 45°, i.s., <A=45°. A guy wire is to be attached to the top of the tower (point P) and to the ground point C, and point C is 200 m downhill from the base of the tower (point B). The angle α in the figure is measured to be 15°. The length of the guy wire should be (A) 282m (B) 200m (C) 346m (D) 141m (E) none of the above A



36. ☐ If $\sin\theta-\cos\theta=\frac{1}{\sqrt{3}}$, and sinθ and cosθ are the two roots to the equation $2x^2+px+q=0$, the value of p^2-8q would be (A) 1/4 (B) 1 (C) 1/3 (D) 3/4 (E) 4/3 E

37. ☐ On a day when the sun passes directly overhead at noon, a 1.80-m-tall man casts a shadow of length $S(t)=1.8|\cot(\pi t/12)|$, where S is the length of shadow in meters and t is the number of hours since 6am, at what time from the following list the length of the shadow would equal to the man's height? (A) 3pm (B) 4pm (C) 5pm (D) 6pm (E) never happen A

38. ☐ Suppose the radius of the circle at right is 6, and the central angle is 120°, then the area of the shaded region is (A) $18\pi-9\sqrt{3}$ (B) $12\pi-12\sqrt{3}$ (C) $9\pi-9\sqrt{3}$ (D) $12\pi-9\sqrt{3}$ (E) none of the above D



39. ☐ If cosθ=tanθ, then the value of $2\sin\theta+\sin^2\theta+\sin^3\theta+\sin^4\theta$ is (A) 2 (B) 3 (C) 1 (D) 0 (E) none of the above A

40. ☐ A hot-air balloon is rising vertically at a constant speed, an observer at a distant observes the elevation angle to be 30° at 10:00am, at 10:10am the elevation angle becomes 34°, then at 10:30am the elevation angle of the balloon should be closest to (using the table below) (A) 34° (B) 39° (C) 41° (D) 42° (E) 43° C

θ	34°	39°	40°	41°	42°	43°
sinθ	0.559	0.629	0.643	0.656	0.669	0.682
cosθ	0.829	0.777	0.766	0.755	0.743	0.731
tanθ	0.675	0.810	0.839	0.869	0.900	0.933