Final Evaluation Problem and Answer Sheets

Section:_____ Name: _____

Write your answer **in the box**. ***Some formulae are provided in the last page.***

1. ☐ What amount of a 60% acid solution must be mixed with a 40% solution to produce 300 mL of a 50% solution? (A)100 mL (B) 125 mL (C) 150 mL (D) 175 mL (E) 200 mL Ans: C

2. ☐ Find the area of the triangle formed by the coordinate axes and the line $2y + 3x - 12 = 0$.
(A) 10 (B) 12 (C) 14 (D) 16 (E) 20 Ans: B

3. ☐ A small business buys a computer for $2400. After 4 years the value of the computer is expected to be $400. For accounting purposes the business uses linear depreciation to assess the value of the computer at a given time t. This means that if V is the value of the computer at time t, then a linear equation is used to relate V and t. Find the depreciated value of the computer 2 years from the date of purchase.
(A) $1100 (B) $1200 (C) $1300 (D) $1400 (E) $1500 Ans: D

4. ☐ The lift of on an airplane wing at takeoff varies jointly as the square of the speed S of the plane and the area A of its wings. A plane with a wing area of 50 m^2 traveling at 60 km/h experiences a lift of 7200 N. How much lift would a plane with a wing area of 100 m^2 traveling at 60 km/h experience?
(A) 12400 N (B) 13400 N (C) 14400 N (D) 15400 N (E) 16400 N Ans: C

5. ☐ A function is given. $f(x) = x^2$, and the indicated transformations are applied to its graph in the given order: stretched vertically by a factor of 3, shifted upward 2 units, and shifted 3 units to the left. What is the equation after the final transformation? (A) $y = -3(x+3)^2 + 2$ (B) $y = 3(x+3)^2 + 2$ (C) $y = 3(x-3)^2 - 2$ (D) $y = -3(x-3)^2 - 2$ (E) $y = -3(x+3)^2 - 2$ Ans: B

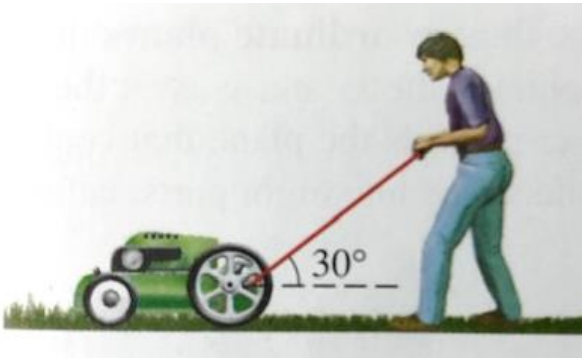
6. ☐ A straight river flows east at a speed of 5 km/h. A motorboat has a speed of 20 km/h relative to the river. A boater wants to arrive at a point on the north shore of the river directly opposite to the starting point. In what direction of northwest should the boat be headed?
(A) $\sin^{-1}(1/2)$ (B) $\sin^{-1}(1/3)$ (C) $\sin^{-1}(1/4)$ (D) $\sin^{-1}(1/5)$ (E) none of the above Ans: C

7. ☐ Find the trace of the sphere $(x-1)^2 + (y-2)^2 + (z-3)^2 = 36$ in the xz- plane.
(A) $(x-1)^2 + (y-2)^2 = 16$ (B) $(y-2)^2 + (z-3)^2 = 16$ (C) $(x-1)^2 + (z-3)^2 = 16$ (D) $(x-1)^2 + (z-3)^2 = 32$ (E) $(y-2)^2 + (z-3)^2 = 32$ Ans: D

8. ☐ Find the direction angles of a vector $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.
(A) $\cos\alpha = (1/\sqrt{10}),\ \cos\beta = (2/\sqrt{10}),\ \cos\gamma = (3/\sqrt{10})$ (B) $\cos\alpha = (1/\sqrt{11}),\ \cos\beta = (2/\sqrt{11}),\ \cos\gamma = (3/\sqrt{11})$ (C) $\cos\alpha = (1/\sqrt{12}),\ \cos\beta = (2/\sqrt{12}),\ \cos\gamma = (3/\sqrt{12})$ (D) $\cos\alpha = (1/\sqrt{13}),\ \cos\beta = (2/\sqrt{13}),\ \cos\gamma = (3/\sqrt{13})$ (E) $\cos\alpha = (1/\sqrt{14}),\ \cos\beta = (2/\sqrt{14}),\ \cos\gamma = (3/\sqrt{14})$ Ans: E

9. ☐ A lawn mower is pushed a distance of 10 m along a horizontal path by a constant force of 100 N. The handle of the lawn mower is held at an angle of 30 degrees from the horizontal, as shown in the figure. Find the work done.

(A) 600 N-m (B) 666 N-m (C) 766 N-m (D) 866 N-m (E) 966 N-m Ans: D



10. ☐ Find the area of a triangle ΔPQR, formed by P(1, 0, 1), Q(0, 1, 0), R(2, 3, 4).

(A) $\sqrt{11}$ (B) $\sqrt{12}$ (C) $\sqrt{13}$ (D) $\sqrt{14}$ (E) $\sqrt{15}$ Ans: D

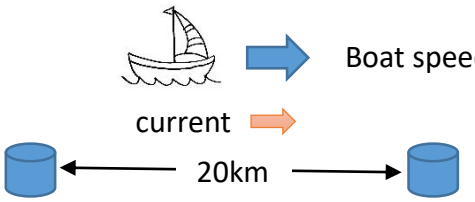
(Hint: The area of a triangle is half the area of a parallelogram, which is the length of the cross product **U** x **V**)

11. ☐ A description of a line is given. The line crosses the z-axis where z = 5 and crosses the xy-plane where x = 2, y = 3. Find the parametric equations for the line.

(A) $x = 2t, y = 3t, z = 5 - 5t$ (B) $x = -2t, y = -3t, z = 5 - 5t$ (C) $x = -2t, y = -3t, z = -5t$
(D) $x = 2t, y = 3t, z = 5 + 5t$ (E) $x = 2t, y = 3t, z = -5t$ Ans: A

12. ☐ A boat on a river travels downstream between two points, 20 km apart, in 1 hour. The return trip against the current takes 2 $^1/_2$ hours. How fast does the current in the river flow?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6 km/h Ans: E



13. ☐ Find the complete solution of the linear system. $\begin{cases} x + y + z = 0 \\ -x + 2y + 5z = 3 \\ 3x - y = 6 \end{cases}$

(A) (1, 3, 2) (B) (-1, 3, 2) (C) (-1, -3, 2) (D) (1, 3, -2) (E) (1, -3, 2) Ans: E

14. ☐ The matrices **A** and **H** are defined as follows: $\mathbf{A} = \begin{bmatrix} 2 & -5 \\ 0 & 7 \end{bmatrix}, \mathbf{H} = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$.

What is the result of the product **A H**?

(A) $\begin{bmatrix} -4 & 7 \\ 14 & 7 \end{bmatrix}$ (B) $\begin{bmatrix} -4 & -7 \\ 14 & 7 \end{bmatrix}$ (C) $\begin{bmatrix} -4 & 7 \\ -14 & 7 \end{bmatrix}$ (D) $\begin{bmatrix} -4 & 7 \\ 14 & -7 \end{bmatrix}$ (E) $\begin{bmatrix} 4 & 7 \\ 14 & -7 \end{bmatrix}$ Ans: D

15. ☐ Find the complete solution of the linear system. $\begin{cases} 3x + y = 2 \\ -4x + 3y + z = 4 \\ 2x + 5y + z = 0 \end{cases}$

(A) (-1, 5, 2) (B) (-1, 5, -15) (C) (-1, -5, 10) (D) (-1, 5, -10) (E) There is no solution Ans: E

16. ☐ The matrix A is defined as $\mathbf{A} = \begin{bmatrix} 6 & -3 \\ 2 & 3 \end{bmatrix}$. Evaluate the determinant $|\mathbf{A}|$.

(A) 12 (B) 16 (C) 18 (D) 24 (E) -12 Ans: D

17. ☐ The matrix A is defined as $\mathbf{A} = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{bmatrix}$. Evaluate the determinant $|\mathbf{A}|$.

(A) -32 (B) -36 (C) -40 (D) -44 (E) -48 Ans: D

18. ☐ The matrix A is defined as $\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$. Find the inverse of the matrix, $\mathbf{A^{-1}}$.

(A) $\frac{1}{2}\begin{bmatrix} -1 & -1 \\ 0 & 2 \end{bmatrix}$ (B) $\frac{1}{2}\begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$ (C) $\frac{1}{2}\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ (D) $\frac{1}{2}\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$ (E) $\frac{1}{2}\begin{bmatrix} -2 & -1 \\ 0 & 1 \end{bmatrix}$ Ans: C

19. ☐ The matrix A is defined as $\mathbf{A} = \begin{bmatrix} 2 & 4 & 1 \\ -1 & 1 & -1 \\ 1 & 4 & 0 \end{bmatrix}$. Find the inverse of the matrix, $\mathbf{A^{-1}}$.

(A) $\begin{bmatrix} 2 & -4 & 1 \\ -2 & 1 & -1 \\ -2 & 4 & 6 \end{bmatrix}$ (B) $\begin{bmatrix} -4 & -4 & 5 \\ 1 & 1 & -1 \\ 5 & 4 & -6 \end{bmatrix}$ (C) $\begin{bmatrix} -2 & 4 & 5 \\ -1 & 1 & 1 \\ 6 & 4 & 6 \end{bmatrix}$ (D) $\begin{bmatrix} -2 & -4 & 6 \\ 1 & 1 & 1 \\ -4 & 5 & 6 \end{bmatrix}$ (E) There is no inverse matrix Ans: B

20. ☐ The partial fraction decomposition of a rational function can be written as

$$\frac{2x^2 - x + 4}{x^3 + 4x} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$$

The solution for the coefficients A, B, and C is

(A) (1, 1, 1) (B) (-1, 1, 1) (C) (1, -1, 1) (D) (1, 1, -1) (E) (1, -1, -1) Ans: D

21. ☐ After simplification, the expression $\frac{\tan\theta - \cot\theta}{\tan^2\theta - \cot^2\theta}$ can be reduced to (A) $\sec\theta\csc\theta$ (B) $\sin\theta\cos\theta$ (C) 1 (D) $\sin\theta + \cos\theta$ (E) none of the above Ans: B

22. ☐ The trigonometric expression $(\sin\theta - \tan\theta)(\cos\theta - \cot\theta)$ can be shown to be identical to which of the followings? (A) $\frac{1}{2}\sin 2\theta - 1$ (B) 1 (C) $(\sin\theta - 1)(\cos\theta - 1)$ (D) $\sin\theta\cos\theta - 1$ (E) none of the above Ans: C

23. ☐ The exact value of the expression $\tan\left(\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{1}{2}\right)$ is (A) $\frac{3\sqrt{3}-4}{4\sqrt{3}+3}$ (B) $\frac{3-4\sqrt{3}}{4+3\sqrt{3}}$ (C) $\frac{3\sqrt{3}+4}{4\sqrt{3}-3}$ (D) $\frac{1}{\sqrt{3}}$ (E) none of the above Ans: B

24. ☐ If θ is in Quadrant IV and $cos\theta = \frac{3}{5}$, ϕ is in Quadrant III and $tan\phi = \sqrt{3}$, then the value of $\cos(\theta - \phi)$ should be (A) $\frac{-4\sqrt{3}+3}{10}$ (B) $-\frac{4\sqrt{3}+3}{10}$ (C) $\frac{4\sqrt{3}+3}{10}$ (D) $\frac{4\sqrt{3}-3}{10}$ (E) none of the above Ans: D

25. ☐ The expression $\frac{1-cos4\theta}{sin4\theta}$ can be simplified to be (A) *2* (B) $2sin2\theta$ (C) $-2sin2\theta$ (D) $tan2\theta$ (E) $-tan2\theta$ Ans: D

26. ☐ The value of sin75°+sin15° is (A) $\frac{\sqrt{3}}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{6}}{2}$ (D) $-\frac{\sqrt{6}}{2}$ (E) 1 Ans: C

27. ☐ The expression $\frac{sinx+sin2x+sin3x}{cosx+cos2x+cos3x}$ can be reduced to (A) $\frac{2cosx+1}{2cosx-1}$ (B) *tan2x* (C) *tan4x* (D) 1 (E) none of the above Ans: B

28. ☐ Which of the followings is NOT a solution to the trigonometric equation $tan\theta sin\theta + sin\theta = 0$? (A) π (B) 0 (C) 3π/4 (D) π/2 (E) 4π Ans: D

29. ☐ The solutions to the equation $cos2\theta - cos\theta = 0$ include (A) $\theta = -\pi/3$ (B) $\theta = \pi/2$ (C) $\theta = \pi$ (D) $\theta = 2\pi/3$ (E) $\theta = -2\pi/3$ Ans: D or E

30. ☐ If θ is in Quadrant II and $cos\theta + sin\theta = \frac{1}{5}$, the value of $\frac{sec\theta}{tan\theta} + \frac{csc\theta}{cot\theta}$ would be (A) 3/5 (B) -5/12 (C) 5/12 (D) 1 (E) none of the above Ans: B

31. ☐ The distance between the two points on the polar plane, $P_1(3,\pi/2)$ and $P_2(-2,7\pi/6)$, is (A) $\sqrt{7}$ (B) $\sqrt{20}$ (C) $\sqrt{19}$ (D) 5 (E) 1 (hint: a formula provided in the last page might help) Ans: A

32. ☐ Regarding the symmetric characteristics of the polar graph of the function $r=5sec\theta$, which of the followings is valid about the graph? (A) symmetric about the pole (B) does not have any symmetric feature (C) symmetric about the $\theta=\pi/2$ axis (D) not symmetric about the polar axis (E) symmetric about the polar axis Ans: E

33. ☐ Let $i = \sqrt{-1}$, the value of the expression $(-1+i)^{-10}$ is (A) $\frac{i}{32}$ (B) $\frac{1}{32}$ (C) $\frac{1+i}{32}$ (D) $\frac{-1+i}{32}$ (E) $\frac{-1+i}{16}$ Ans: A

34. ☐ The solution to the logarithmic inequality $log_{\frac{1}{3}}\left(log_8(10-x)\right) > 1$ is (A) $3<x<8$ (B) $8<x<9$ (C) $6<x<10$ (D) $8<x<10$ (E) none of the above Ans: B

35. ☐ For $\triangle ABC$ with three sides *a, b,* and *c*, if $a=4$ and $b=3$, then the value of *c* which yields max area is (A) 3 (B) 4 (C) 5 (D) 6 (E) 8 Ans: C

36. ☐ If sinα and cosα are the two roots to the equation $4x^2-5x+a=0$, then the value of *a* should be (A) $\frac{1}{8}$ (B) $\frac{9}{8}$ (C) $-\frac{1}{8}$ (D) $-\frac{9}{8}$ (E) $\frac{3}{8}$ Ans: B

37. ☐ If $\cos 2\alpha=\frac{1}{8}$, and 2α is in Quadrant IV, then $\cos\alpha$= (A) $-\frac{9}{8}$ (B) $\frac{9}{8}$ (C) $-\frac{3}{4}$ (D) $\frac{3}{4}$ (E) $-\frac{\sqrt{14}}{4}$ Ans: C

38. ☐ If we convert the rectangular equation $2y=x^2$ into polar form, the result will be (A) $r=\tan\theta\sec\theta$ (B) $r=\cos^2\theta\sin\theta$ (C) $r=2\tan\theta\sec\theta$ (D) $r=2\cos^2\theta\sin\theta$ (E) none of the above Ans: C

39. ☐ $\sin(350°+\theta)\cos(110°+\theta)-\cos(350°+\theta)\sin(110°+\theta)$= (A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $-\frac{\sqrt{3}}{2}$ (E) none of the above Ans: D

40. ☐ Which of the followings yields the largest value? (A) sin(-1120°) (B) tan(-250°) (C) cot80° (D) csc240° (E) cos580° Ans: C

Note: Some formulae for your references, derive the others when needed

Addition/subtraction formulae: $sin(\alpha \pm \beta) = sin\alpha cos\beta \pm cos\alpha sin\beta$

$$cos(\alpha \pm \beta) = cos\alpha cos\beta \mp sin\alpha sin\beta$$

Half angle formulae: $sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-cos\alpha}{2}}, cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+cos\alpha}{2}}$

Product-to-sum: $sin\alpha cos\beta = \frac{1}{2}[sin(\alpha+\beta) + sin(\alpha-\beta)]$

$$cos\alpha cos\beta = \frac{1}{2}[cos(\alpha+\beta) + cos(\alpha-\beta)]$$

Distance formula in polar coordinate: $d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2cos(\theta_2-\theta_1)}$

2X2 Matrix inverse formula: $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$