# Examination Papers of the 2021 Entrance Exam, UPTP

- ***Write your answers in the answer sheet.***
- ***Some formulae are provided in the last page. (Page6)***

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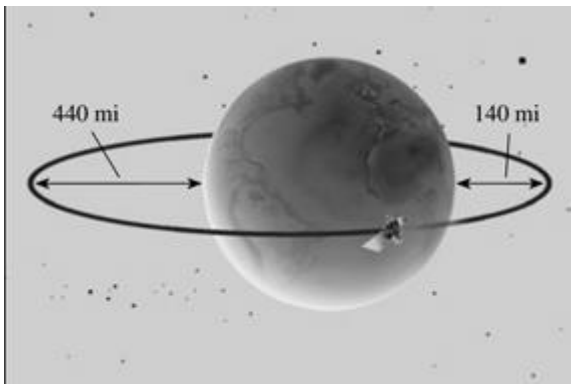
1. A satellite is in an orbit around the earth with the center of the earth at one focus as shown in the figure.




Which kind of conic curve do we have to learn?
(A) Circles (B) Ellipses (C) Parabolas (D) Hyperbolas (E) two intersect lines.

2. What is the minimum value of the given function?
$f(x) = 2x^2 - 12x + k,\ 2 \le x \le 10,\ k$ is a constant.
(A) $f(3)$ (B) $f(2)$ (C) $f(10)$ (D) $f(2) + k$ (E) none of the above.

3. The *n*th term of an arithmetic sequence is given by $a_n = a + (n-1)d$. Find the sum of the first 10 numbers if $a = 5$ and $d = 3$.
(A) 175 (B) 360 (C) 570 (D) 650 (E) none of the above.

4. Find the sum $\sum_{k=1}^{6} ar^{k+1}$, where *a*=1 and *r*=3.

$$\sum_{k=1}^{6} ar^{k+1} = 9 + 27 + \cdots + 2187$$

(A) 3274 (B) 3275 (C) 3276 (D) 3277 (E) none of the above.

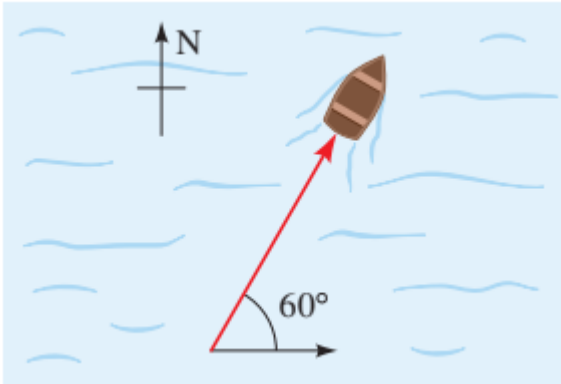
5. Solve the nonlinear inequality $x^2 > x + 12$.
(A) $-2 < x < 6$ (B) $x < -3$ or $4 < x$ (C) $2 < x < 6$ (D) $x < -4$ or $3 < x$ (E) none of the above.

6. The matrix A is defined as $\mathbf{A} = \begin{bmatrix} 2 & 0 & 5 \\ -2 & -3 & 2 \\ 3 & 0 & 3 \end{bmatrix}$. Evaluate the determinant $|\mathbf{A}|$.

(A) 26 (B) 27 (C) 28 (D) 29 (E) -10.

7. A polynomial is defined as:
$p(x) = 2x^3 - 12x^2 + 22x - 12$
What are the zeros of the polynomial?
(A) 1, 2, 3 (B) -1, -2, -3 (C) 2, 4, 6 (D) -2, -4, -6 (E) none of above.

8. A straight river flows east at a speed of 20 km/h. boater starts at the south shore of the river and heads in a direction $60^0$ from the shore (see the figure). The motorboat has a speed of 10 km/h. relative to the water. Find the true velocity of the motorboat.
(A) $25\mathrm{i} + 5\sqrt{3}j$ (B) $20\mathrm{i} + 10\sqrt{3}j$ (C) $25\mathrm{i} + 10\sqrt{3}j$ (D) $10\mathrm{i} + 5\sqrt{3}j$ (E) none of the above.




9. The perception of the loudness B (in decibels, dB) of a sound with physical intensity $I$ (in W/m$^2$) is given by $B = 10 \log (I/I_0)$. If $I$ is 3,000,000 times that of $I_0$, find the decibels of a sound.
(Hint: $\log_{10}2=0.301$, $\log_{10}3=0.477$, $\log_{10}5=0.699$)
(A) -60 (B) 30 (C) 67 (D) 50 (E) -50.

10. To divide the function $f(x\text{-}1)$ by $(x^2+2x\text{-}3)$, we find that the quotient is $Q(x)$ and the remainder is $(x\text{-}2)$. If $f(0)=a$ and $f(\text{-}4)=b$, what is the value of $b/a$?
(A) 5 (B) 4 (C) 3 (D) 2 (E) none of the above.

11. Find the area of a triangle ΔPQR, formed by P(2, 0, 2), Q(0, 2, 0), R(1, 2, 3).
(A) $\sqrt{12}$ (B) $\sqrt{13}$ (C) $\sqrt{14}$ (D) $\sqrt{15}$ (E) 4.
(Hint: The area of a triangle is half the area of a parallelogram, which is the length of the cross product **u** x **v**)

12. The value of $\log_9 0.24$ can be rounded to two decimal places. What is the approximate value?
(Hint: $\log_{10}2=0.301$, $\log_{10}3=0.477$)
(A) -0.79 (B) -0.65 (C) 0.65 (D) 0.82 (E) 0.79.

13. The matrix A is defined as $\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 1 & 2 \end{bmatrix}$.
Find the inverse of the matrix, $\mathbf{A}^{-1}$.
(A) $\begin{bmatrix} 3 & -2 & 1 \\ -1 & 1 & 1 \\ 1 & 4 & 6 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & -2 & -5 \\ -1 & 1 & 1 \\ -3 & 2 & 6 \end{bmatrix}$ (C) $\begin{bmatrix} 3 & -2 & 5 \\ 1 & 1 & 1 \\ -3 & -2 & 6 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & -2 & 1 \\ 1 & 1 & 1 \\ -3 & 3 & 6 \end{bmatrix}$
(E) There is no inverse matrix.

14. Find $\sin\theta$ between two vectors **v** = (4, -3) and **u** = (3, 2).
(A) $\frac{6}{5\sqrt{13}}$ (B) $\frac{17}{5\sqrt{13}}$ (C) $\frac{5}{7}$ (D) 0.8 (E) none of the above.

15. Vectors **v** = (-1, 3, 3) and **u** = (5, 4, 1), Calculate the product of **(u·v)(u×v)**.
(A) -90**i**+190**j**+20**k** (B) 30**i**+60**j**-160**k** (C) -90**i**+160**j**-190**k** (D) -30**i**-20**j**-90**k** (E) none of the above.

16. Calculate the cross-sectional area intersected by a sphere $((x+a)^2 + (y-b)^2 + (z+1)^2 = 25)$ and a plane $z = 3$.
(A) $3\pi$ (B) $4a\pi$ (C) $9\pi$ (D) $16\pi$ (E) $25\pi$.

17. Find a line that is parallel to the line $6x + 4y = 5$ and passes through the origin.
(A) $6x - 4y = 0$ (B) $3x + 2y = 5$ (C) $3x - 2y = 0$ (D) $3x + 2y = 0$ (E) none of the above.

18. What amount of a 60% acid solution must be mixed with a 20% solution to produce 500 mL of a 30% solution?
(A) 100 mL (B) 125 mL (C) 150 mL (D) 175 mL (E) 200 mL.

19. Find the area of the triangle formed by the coordinate axes and the line 3y + 5x + 30 = 0.
(A) 50 (B) 30 (C) 15 (D) 100 (E) none of the above.

20. Calculate $\cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{7\pi}{4}\right)$.

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{2\pi}{3}$ (D) $\pi$ (E) $\frac{7\pi}{4}$.

21. For a circle with radius of 4, find the terminal point determined by $\frac{2\pi}{3}$.

(A) (4, 0) (B) (-2, $2\sqrt{3}$) (C) ($2\sqrt{3}$, 2) (D) ($\frac{-\sqrt{2}}{2}$, $\frac{\sqrt{2}}{2}$) (E) ($\frac{-\sqrt{2}}{2}$, $\frac{-\sqrt{2}}{2}$).

22. A function is given.

$$f(x) = \frac{x-3}{x+3}$$

What is the inverse function of $f(x)$?

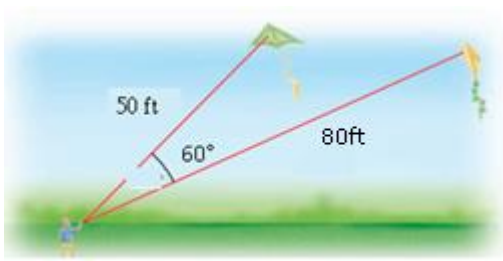
(A) $f^{-1}(x) = \frac{-3(x+1)}{x-1}$ (B) $f^{-1}(x) = \frac{3(x+1)}{x-1}$ (C) $f^{-1}(x) = \frac{-3(x-1)}{x+1}$ (D) $f^{-1}(x) = \frac{3(x-1)}{x+1}$ (E) none of the above.

23. A function is given,
$2x^2 - 4x + y + 4 = 0$,
What is the indicated transformations of $x^2 = 4py$ are applied to its graph in the given order?
(A) shifted upward 1 unit, and shifted 2 units to the left. (B) shifted downward 2 units, and shifted 1 unit to the right. (C) shifted downward 3 units, and shifted 2 units to the right. (D) shifted upward 3 units, and shifted 3 units to the right. (E) none of the above.

24. Use the following functions: $f(x) = x - 3$ and $g(x) = 3x^2 + 1$,
to evaluate the composition of functions (g ◦ f)(-2) + (f ◦ g)(-2).
(A) 43 (B) -34 (C) 86 (D) -86 (E) none of the above.

25. Find the area of this region in the coordinate plane that satisfies both the inequalities $x^2 + y^2 \leq 8$ and $y \geq |x|$.
(A) 2π (B) 4π (C) 6π (D) 8π (E)10π.

26. A boy is flying two kites at the same time. He has 50 ft of line out to one kite and 80 ft to the other. He estimates the angle between the two lines to be $60^0$.Find the distance between the kites.
(A) 60 (B) 70 (C) 80 (D) 90 (E)100 ft.



27. A plane intersects the coordinate axes at the points (2, 0, 0), (0, 3, 0), and (0, 0, 4). The obtained plane equation is $ax + by + cz + d = 0$. What is the value of $d$ ?
(A) $-1$ (B) $-2$ (C) $-3$ (D) $-6$ (E) none of the above.

28. A rational function is defined as:
$$r(x) = \frac{x^3 + 2x + 3}{x^2 - x - 1}$$
What is the slant asymptotes of $r(x)$?
(A) x (B) x+1 (C) x+2 (D) x+3 (E) none of the above.

29. Find the area of the triangle whose sides have the lengths a=6, b=8, c=10.
(A) 6 (B) 12 (C) 18 (D) 24 (E) 32.

30. If sinθ = cotθ, then the value of $\cos\theta + 2\cos^2\theta + 2\cos^3\theta + \cos^4\theta$ is
(A) 2 (B) 3 (C) 1 (D) 0 (E) none of the above.

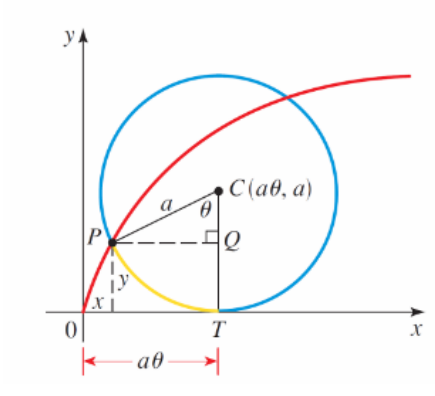
31. Find the value of $\sin\ 75^\circ + \cos\ 75^\circ$.
(Hint: $\sin\ 30^\circ = \cos\ 60^\circ = \frac{1}{2}; \sin\ 45^\circ = \cos\ 45^\circ = \frac{\sqrt{2}}{2}; \sin\ 60^\circ = \cos\ 30^\circ = \frac{\sqrt{3}}{2}$)
(A) $\frac{\sqrt{6}}{2}$ (B) $\frac{-\sqrt{6}}{2}$ (C) $\frac{-\sqrt{2}}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) none of the above.

32. The function $\tan\left(x - \frac{\pi}{4}\right)$ is equivalent to the expression of
(A) $\frac{-1+\tan x}{1+\tan x}$ (B) $\frac{1+\tan x}{-1+\tan x}$ (C) $\frac{1-\tan x}{1+\tan x}$ (D) $\frac{\tan x-1}{\tan x+1}$ (E) none of the above.

33. Two equations $x = 2t^2 - 5t$ and $y = \frac{t}{3}$. Which one of the following equations is correct for equation $x$?
(A) $x = -2y^2 + 20y$ (B) $x = 21y^2 - 10y$ (C) $x = 12y^2 + 10y$ (D) $x = 18y^2 - 15y$ (E) none of the above.

34. Find the rectangular coordinates for the point that has $(2, 150^\circ)$ in polar coordinates.
(Hint: $\sin\ 30^\circ = \cos\ 60^\circ = \frac{1}{2}; \sin\ 45^\circ = \cos\ 45^\circ = \frac{\sqrt{2}}{2}; \sin\ 60^\circ = \cos\ 30^\circ = \frac{\sqrt{3}}{2}$)
(A) $(-\sqrt{3}, 1)$ (B) $(-\sqrt{3}, 2)$ (C) $(2, -2\sqrt{3})$ (D) $(-2, -2\sqrt{3})$ (E) none of the above.

35. A cycloid curve is defined by the figure. Please find the coordinates $x$, $y$ in terms of the radius $a$ and the angle $\theta$.




(A) $y = a(1 - \sin\theta)$ (B) $x = a(\theta - \sin\theta)$ (C) $x = a(1 - \cos\theta)$ (D) $y = a(\theta - \cos\theta)$ (E) none of the above.

36. Two points on the polar plane are given as $P_1 = (-1, -\pi/2)$ and $P_2 = (1, -2\pi/3)$. Please find the triangular area formed by $P_1$, $P_2$, and the pole.

(A) $1/2$ (B) $1/4$ (C) $\sqrt{2}$ (D) $\sqrt{3}$ (E) none of the above.

37. Find the partial fraction decomposition of the rational function.

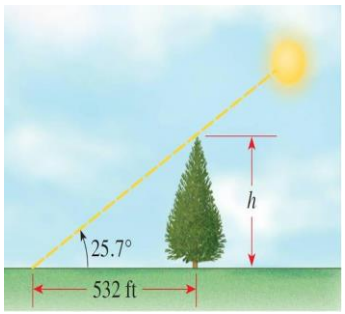
$\frac{x^2+x+3}{x^2+x-2} = P(x) + \frac{A}{x+2} + \frac{B}{x-1}$, where $P$ is a polynomial and A and B are two constants. What is the value of A?

(A) $A = \frac{-1}{3}$ (B) $A = \frac{4}{3}$ (C) $A = \frac{-5}{3}$ (D) $A = \frac{10}{3}$ (E) none of the above.

38. Matrices **A** and **B** are defined as $\mathbf{A} = \begin{bmatrix} 0.25 & 0.3 & 0.2 \\ 0.5 & 0.5 & 0.4 \\ 0.25 & 0.2 & 0.4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 500 & 200 \\ 200 & 400 \\ 500 & 100 \end{bmatrix}$. Solve **BA**.

(A) $\begin{bmatrix} 570 & 150 \\ 700 & 370 \\ 430 & 280 \end{bmatrix}$ (B) $\begin{bmatrix} 255 & 150 \\ 750 & 250 \\ 155 & 210 \end{bmatrix}$ (C) $\begin{bmatrix} 360 & 150 \\ 260 & 380 \\ 545 & 480 \end{bmatrix}$ (D) $\begin{bmatrix} 285 & 190 \\ 550 & 340 \\ 365 & 170 \end{bmatrix}$ (E) none of the above.

39. A giant redwood tree casts a shadow, as shown in the figure. (Hint: $\tan 25.7^\circ = 0.4817$)




(A) If the shadow casted is 532 ft long, and if the angle of elevation of the sun is 25.7°, then the height of the tree approximately equals to 378 ft.

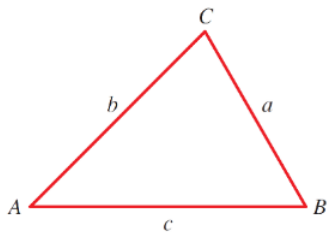
(B) If the angle of elevation of the sun is 48.5°, the height of the tree is 256 ft, then the shadow casted is approximately 312 ft.

(C) If the angle of elevation of the sun is 36.8°, the shadow casted is 312 ft then the height of the tree is approximately 432 ft.

(D) None of the above (A) (B) (C) is true.

(E) All the above (A) (B) (C) are true.

40. To solve triangle ABC(with angles A,B and C, sides a, b, and c, with side length la, lb, lc respectively)




(A) If $\angle A = 45°$, $l_a = 7\sqrt{2}$, and $l_b = 7$, then there is one solution.

(B) If $\angle A = 30°$, $l_a < l_b/2$, then there is no solution.

(C) If $\angle A = 30°$, $l_a > l_b/2$, then there are two solutions.

(D) All the above (A) (B) (C) are true.

(E) None of the above (A) (B) (C) is true.

***Some formulae are provided in the last page.(Page6)***