Examination Paper of the 2026 Entrance Exam, UPTP

General Instructions

- The exam duration is **2 hours**, starting from the beginning of the exam.
- **Write your answers on the answer sheet**. Answers written on this examination paper will not be considered for grading.
- Some formulae are provided on the last page.
- The scoring policy is as follows:
  - The maximum possible score is **100 points**.
  - All students start with **20 points**.
  - Each correct response adds **2 points**.
  - Each incorrect response subtracts **0.5 points**.
  - Unanswered questions do not affect the score.
- Notation:
  - Vectors are denoted using arrow notation, e.g., $\vec{v}$.
  - The standard unit vectors are denoted by $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.
  - Matrices are denoted using bold notation, e.g., $\mathbf{A}$.
  - $\mathbb{R}$ denotes the set of real numbers.
  - ln denotes the natural logarithm, log denotes the common logarithm (base–10), and in general $\log_a$ denotes the logarithm with base a.

1. Which of the following is the factorization of the polynomial $5x(5x-2)-4(5x-2)^2$?

   A. $(5x-2)(-15x+8)$

   B. $(5x-2)(25x-8)$

   C. $(5x-2)(5x-4)$

   D. $(5x-2)(-20x+4)$

   E. $(5x-2)(-15x-8)$

2. Consider the following equation of a non-degenerate shifted conic

$$Ax^2+Cy^2+Dx+Ey+F=0,$$

   where $A, C, D, E, F \in \mathbb{R}$ and A and C are not both zero. Choose the correct answer:

   A. If A or C is 0, the resulting equation corresponds to a hyperbola.

   B. If A or C is 0, the resulting equation corresponds to a parabola that passes through the origin.

   C. If A and C have opposite signs, the equation corresponds to a hyperbola.

   D. If A and C have the same signs, the equation corresponds to a circle.

   E. None of the above choices are correct.

3. Simplify $\frac{9}{(4-\sqrt{7})}$ into the form $a+b\sqrt{7}$, where a and b are integers. What is the value of $a+b$?

   A. 3 **B.** 5 C. -9 D. -15 E. 15

4. The point $P = (1, 2)$ is moved 2 units in the positive x-direction and 1 unit in the negative y-direction. The vector defining the new position of the point P is:

   A. $2\hat{\imath} + 3\hat{\jmath}$ B. $\hat{\imath} + \hat{\jmath}$ C. $3\hat{\imath} + 3\hat{\jmath}$ D. $\hat{\imath} + 3\hat{\jmath}$ **E.** $3\hat{\imath} + \hat{\jmath}$

5. Let $y = x + 2$. If
$$\lim_{x\to 3} y = c,$$
then the value of c is:

   A. 3 B. 4 C. 6 **D.** 5 E. 7

6. Suppose $\sin(\theta) + \cos(\theta) = \frac{\sqrt{6}}{2}$. What is the smallest non-negative value of θ?

   A. $60°$ **B.** $15°$ C. $30°$ D. $75°$ E. $45°$

7. The value of $\cos(1°) + \cos(2°) + \cos(3°) + \cdots + \cos(360°)$ is:

   A. 0 B. -1 C. $-\frac{1}{2}$ D. $\frac{1}{2}$ E. 1

8. Let $y = \ln x$. The values of y that satisfy the equation $e^{5y^2+3} = x^8$ are:

   A. $y = 1$ or $y = -3/5$ B. $y = -1$ or $y = 3/5$ C. $y = \frac{8\pm\sqrt{34}}{10}$ D. $y = \ln 8$ **E.** $y = 1$ **or** $y = 3/5$

9. Let L_1 be the line $5x + 3y = 5$ and let L_2 be the line $3x + 2y = 6 - 2a$, where a is a real number. The x-coordinate of the intersection point of L_1 and L_2 is:

   A. $6a + 8$ B. $8 - 6a$ **C.** $6a - 8$ D. $\frac{6a-8}{5}$ E. $2a - 8$

10. Let $\vec{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{b} = \lambda\hat{\imath} + 2\hat{k}$ be such that the angle between them is $60°$. The largest real λ is:

    A. $-4 + \sqrt{12}$ B. $4 - \sqrt{12}$ C. $-4 - \sqrt{12}$ **D.** $4 + \sqrt{12}$ E. None are correct.

11. Consider $(3x + 2)(-x^6 + 3x^5) + (3x + 2)(-2x^6 + x^5) + (x + 1)(3x^6 - 4x^5)$. This expression can be equivalently written as:

    A. $(3x^6 - 4x^5)(2x + 1)$

    B. $(3x^6 - 4x^5)(2x + 3)$

    C. $-(3x^6 - 4x^5)(2x + 3)$

    D. $-(3x^6 - 4x^5)(2x + 1)$

    E. $-(3x^6 - 4x^5)(x + 1)$

12. Assuming $x > 0$, the value of $\left(x^{a+b}\right)^{a-b}\left(x^{b+c}\right)^{b-c}\left(x^{c+a}\right)^{c-a}$ is:

    A. 0 **B.** 1 C. -1 D. 2 E. -2

13. A sequence a_n has $a_1 = \frac{1}{2}$ and $a_{n+1} = 2a_n^2 - 1$ for all $n \geq 1$. What is a_{20}?

    A. $-\frac{1}{2}$ B. $\frac{\sqrt{3}}{2}$ C. 0 D. $\frac{1}{2}$ E. $-\frac{\sqrt{3}}{2}$

14. Which of the following expressions is equal to $\sin^3\theta\cos\theta + \sin\theta\cos^3\theta$?

    A. $\sin\theta$

    B. $\sin^2\theta + \cos^2\theta$

    C. $\frac{1}{2}\sin(2\theta)$

    D. $\sin(2\theta)$

    E. $\frac{1}{2}\cos(2\theta)$

15. When $(3x^2 - 3)$ is multiplied by $(x - 1)$ and then divided by $(x + 1)$, the remainder is:

    A. 3 B. 2 C. -1 D. 1 **E.** 0

16. An equivalent expression to $\left(x^2\right)^{\frac{3}{2}}$ is:

    A. x^3 B. x^2 **C.** $|x|^3$ D. $|x|^2$ E. x

17. Among the following matrix multiplications, select the option where the matrix multiplication is defined and the equation is correct. (Note: The terms $[-1]$ and $[-5]$ in the options are both 1×1 square matrices.)

    A. $\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} -1 \end{bmatrix} = \begin{bmatrix} -1 & -2 \end{bmatrix}$

    B. $\begin{bmatrix} -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

    C. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \end{bmatrix} = \begin{bmatrix} 17 & 39 \end{bmatrix}$

    D. $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} -1 & -2 \end{bmatrix} = \begin{bmatrix} -5 \end{bmatrix}$

    E. $\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 109 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -110 \end{bmatrix}$

18. Which of the following matrices is equal to the matrix $\begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}^3$? (The n-th power of a matrix $\mathbf{A}$ is obtained by multiplying the matrix $\mathbf{A}$ by itself n times. For example, $\mathbf{A}^2 = \mathbf{A} \times \mathbf{A}$)

    A. $\begin{bmatrix} -1 & 0 \\ -3 & -1 \end{bmatrix}$ B. $\begin{bmatrix} -1 & 0 \\ -3 & 1 \end{bmatrix}$ C. $\begin{bmatrix} -1 & 3 \\ 0 & -1 \end{bmatrix}$ **D.** $\begin{bmatrix} -1 & 0 \\ 3 & -1 \end{bmatrix}$ E. $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

19. A company has only three members: a Manager, a Secretary, and a Sales Representative. If only the Secretary receives a 10% salary increase, the company's total salary expenditure increases by 3%; if only the Sales Representative receives a 20% salary increase, the company's total salary expenditure increases by 4%. If only the Manager's salary is reduced by 15%, by what percentage will the company's total salary expenditure decrease?

    A. 5.5 % **B.** 7.5 % C. 3.5 % D. 6.5 % E. 4.5 %

20. Let $a_1, a_2, a_3, a_4, a_5 \ldots$ be a geometric sequence with common ratio r. The geometric sequence $a_1, a_3, a_5 \ldots$ has common ratio:

    A. r^2 B. r C. $2r$ D. $\frac{r}{2}$ E. $\sqrt{r}$

21. As shown in the figure, there is a triangle $\triangle ABC$. Let $\overline{AD}$ be the height of the triangle on the side $\overline{BC}$. Given $\overline{AD} = 12$, $\tan \angle B = \frac{3}{2}$ and $\tan \angle C = \frac{2}{3}$. The length $\overline{BC}$ is:

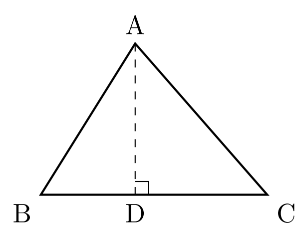
    A. 20

    B. 21

    C. 24

    D. 25

    E. 26



22. The values of x that solve $\ln(x+1) + \ln(3-x) = 0$ are:

    A. $\{1-\sqrt{3}, 1+\sqrt{3}\}$ B. $\{1+\sqrt{3}\}$ C. $\{1-\sqrt{3}\}$ D. $\{-1, 3\}$ E. $\{1, -3\}$

23. Let $a_1, a_2, a_3,$ be a geometric progression with the first term $a_1 = 10$ and the common ratio $r = 10$. Let $b = \log_{a_1} a_2 + \log_{a_2} a_3$. Select the correct option.

    A. $b = \frac{5}{2}$ B. $b = \frac{3}{2}$ C. $b = \log_{10} 3$ **D.** $b = \frac{7}{2}$ E. $b = \log_3 10$

24. The values of a and b such that the system

$$\begin{cases} x - y = 0 \\ 2x + ay = b \end{cases}$$

has no solution are:

A. $a = 2, b = 0$ B. $a = 2, b \neq 0$ **C.** $a = -2, b \neq 0$ D. $a = -2, b = 0$ E. none of the above

25. Let $a \neq 0$. Consider the following system of equations:

$$x + y = a,$$
$$x^2 - y^2 = a.$$

The solution (x, y) of the system is:

A. $(a+1, a-1)$

B. $(\frac{a+1}{2}, \frac{a-1}{2})$

C. $(a, 0)$

D. $(\frac{a-1}{2}, \frac{a+1}{2})$

E. $(a-1, a+1)$

26. The value of x that solves $2^x = 5$ is:

A. $\log_5 2$ **B.** $\log_2 5$ C. $\ln 5$ D. $5/2$ E. $2/5$

27. The point $P = (4, \pi/3)$ is expressed in polar coordinates. The same point P in Cartesian coordinates is:

A. $(-2, 2\sqrt{3})$

B. $(2, -2\sqrt{3})$

C. $(-2, -2\sqrt{3})$

D. $(2, 2\sqrt{3})$

E. $(2, \sqrt{3})$

28. The center C and radius r of the circle $(x-1)^2 + (y+2)^2 = 16$, are:

A. $C = (-1, 2)$, $r = 4$

B. $C = (1, -2)$, $r = 16$

C. $C = (-1, 2)$, $r = 8$

D. $C = (1, -2)$, $r = 8$

E. $C = (1, -2)$, $r = 4$

29. Suppose that the number of terms in an arithmetic progression is $2k$, where k is a positive integer. If the sum of all the odd terms on the arithmetic progression is 40, the sum of all the even terms is 55 and the last term exceeds the first term by 27, then k is equal to:

A. 5 B. 8 C. 6 D. 4 E. 7

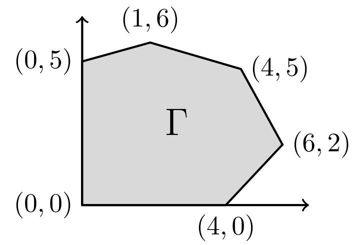
30. A line ℓ has equation $y = 6 - 2x$. A second line is perpendicular to ℓ and passes through the point $(-6, 0)$. The area of the region enclosed by the two lines and the x-axis is:

A. $\frac{81}{2}$ B. 18 **C.** $\frac{81}{5}$ D. $\frac{108}{5}$ E. 27

31. The first three terms of an increasing arithmetic progression are p, q and p^2. The first three terms of a geometric progression are p, p^2 and q. The sum of the first 10 terms of the arithmetic progression is:

A. $\frac{105}{8}$ B. $\frac{23}{8}$ **C.** $\frac{95}{8}$ D. $\frac{115}{8}$ E. $\frac{185}{8}$

32. There is a polygonal region Γ (including its boundary) in the coordinate plane, as shown in the figure. If $k > 0$, the line $y = \dfrac{k}{2} - \dfrac{7x}{2}$ and the two coordinate axes form a triangular region such that the polygonal region Γ lies entirely within this triangular region (including its boundary). The smallest positive real number k is:



A. 50

B. 46

C. 19

D. 38

E. 28

33. Consider

$$\frac{1+3x}{2-x} < 2.$$

The largest set of x satisfying the inequality is:

A. $(-\infty, \frac{3}{5}) \cup (2, \infty)$

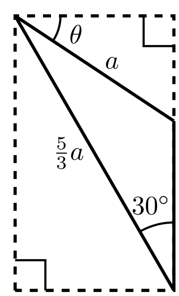
B. $(-\infty, -\frac{3}{5}] \cup (2, \infty)$

C. $(-\infty, -\frac{3}{5}) \cup [2, \infty)$

D. $(-\infty, \frac{3}{5}) \cup [2, \infty)$

E. $(-\infty, -\frac{3}{5}) \cup (-2, \infty)$

34. The angle θ (measured in radians) from the figure below is:



A. $\sin^{-1}(\frac{5}{6})$

B. $\cos^{-1}(\frac{5}{3})$

C. $\cos^{-1}(\frac{\sqrt{3}}{2})$

D. $\tan^{-1}\left(\frac{5}{6}\right)$

E. $\cos^{-1}\left(\frac{5}{6}\right)$

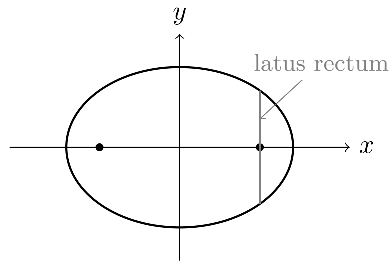
35. The perpendicular distance of the line $\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2}$ from the point $P = (2, -10, 1)$ is:

A. $3\sqrt{2}$ B. 6 C. $5\sqrt{2}$ **D.** $3\sqrt{5}$ E. $4\sqrt{3}$

36. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{3}$. If $\lambda\vec{a} + 2\vec{b}$ and $3\vec{a} - \lambda\vec{b}$ are perpendicular to each other, then the product of all possible real values of λ is

A. -6 B. 6 C. 36 D. $\sqrt{6}$ E. -36

37. A *latus rectum* for an ellipse is a line segment perpendicular to the major axis at a focus, with endpoints on the ellipse. For the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$, the length of the *latus rectum* is:



A. 1

B. $\sqrt{2}$

C. 4

D. 2

E. None of the above.

38. The coefficient of x^8 in the expansion of $(2x + 3x^2)^6$ is equal to the coefficient of x^4 in the expansion of $(1 - ax^2)^5$. Find all possible values of a^2.

A. 226

B. 216

C. 236

D. 246

E. There are no possible values of a.

39. Let $0 < b \leq \frac{1}{2}$ and let $a = 2b$. For which real numbers x does the inequality

$$\left| \frac{ax + b}{ax - b} \right| \geq 1$$

hold for every such b?

A. $(-\infty, 0)$ **B.** $[0, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$ C. $(0, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$ D. $(-\infty, 0) \cup (0, \frac{1}{2})$ E. $\mathbb{R} \setminus \{\frac{1}{2}\}$

40. A set of experimental data can be modeled by the following equation:

$$y = \alpha x^{\beta},$$

where α and β are constants and $x > 0$. If the graph $\ln(y)$ vs. $\ln(x)$ is a straight line with slope -1 and the y-intercept 2, the values of α and β are:

A. $\alpha = 2,\ \beta = -1$

B. $\alpha = e^{-2},\ \beta = -1$

C. $\alpha = e^2,\ \beta = 1$

D. $\alpha = e^2,\ \beta = -1$

E. $\alpha = -2,\ \beta = -1$

Note: Some formulae for your reference, derive others when needed.

- Base of natural logarithm: $e \approx 2.718$.
- Continuously compound interest: $A = Pe^{rt}$, A: yield, P: capital, r: yearly interest rate, t: number of years.
- Addition/subtraction formulae:

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta),$$
$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta).$$

- Half angle formulae:

$$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{2}},$$
$$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos(\alpha)}{2}}.$$

- Double angle formulas:

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$
$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$$

- Product-to-sum:

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}\left(\sin(\alpha+\beta) + \sin(\alpha-\beta)\right)$$
$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left(\cos(\alpha+\beta) + \cos(\alpha-\beta)\right)$$

- Law of Cosines: $a^2 = b^2 + c^2 - 2bc\cos(\angle A)$.
- Distance formula in polar coordinates: $d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$.
- Matrix inverse formula: $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.
- Dot product between two vectors: $\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$.
- Cross product between two vectors: $|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\sin\theta$.
- In a circle of radius r the area A of a sector with a central angle of θ radians is $A = \frac{1}{2}r^2\theta$.
- The area A of a triangle with sides of lengths a and b and with included angle θ is $A = \frac{1}{2}ab\sin(\theta)$.
- The area A of a triangle from the lengths of its three sides a, b, and c is given by: $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$.
- The volume V of the parallelepiped determined by the vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ is $V = \left(\vec{u}\cdot(\vec{v}\times\vec{w})\right)$.
- Standard equation of ellipse: $\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$, where (x_0, y_0) is the center of the ellipse.
- Standard equation of hyperbola: $\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = 1$, where (x_0, y_0) is the center of the hyperbola.
- Rotation of Axes:

$$x = X\cos\phi - Y\sin\phi, \qquad y = X\sin\phi + Y\cos\phi,$$
$$X = x\cos\phi + y\sin\phi, \qquad Y = -x\sin\phi + y\cos\phi.$$

- General conic equation: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, rotate the axes with the angle ϕ that satisfies $\cot(2\phi) = \frac{A-C}{B}$.
- Sum of arithmetic series: $S_N = \sum_{n=1}^{N} a_n = \frac{(a_1+a_N)N}{2}$, with $a_n = a_1 + (n-1)d$, d: common difference.
- Sum of geometric series: $S_N = \sum_{n=1}^{N} a_n = \frac{a_1(r^N-1)}{(r-1)}$, with $a_n = a_1 r^{n-1}$, r: common ratio.
- $\log 2 \approx 0.301$, $\log 3 \approx 0.477$, $\log 4 \approx 0.602$, $\log 5 \approx 0.699$.

Table of notable angles.

θ	$0°$	$30°$	$36.9°$	$45°$	$60°$	$90°$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{4}{5}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{\sqrt{3}}{3}$	$\frac{3}{4}$	1	$\sqrt{3}$	—
$\cot(\theta)$	—	$\sqrt{3}$	$\frac{4}{3}$	1	$\frac{\sqrt{3}}{3}$	0
$\sec(\theta)$	1	$\frac{2\sqrt{3}}{3}$	$\frac{5}{4}$	$\sqrt{2}$	2	—
$\csc(\theta)$	—	2	$\frac{5}{3}$	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1